A Choice-Functional Characterization of Welfarism

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Abstract

Welfarism is the view that individual welfare is the only thing that matters. One of the most important contributions of social choice theory has been to provide a precise formulation and axiomatic characterization of welfarism using Amartya Sen's framework of social welfare functionals. This paper is motivated by the observation that the standard formalization of welfarism is too restrictive, since a welfarist social planner need not be committed to maximizing a preference ordering or any other binary relation over alternatives. We therefore provide a characterization of welfarism in a more general choice-functional setting and show that welfarism, so understood, carries no commitment to rationalizability. This more general characterization is compatible with welfare levels having any structure whatsoever. It also sheds light on different formulations of anonymity, revealing only some of these to be fundamental requirements of impartiality.

1 Introduction

Welfarism is, very roughly, the view that individual welfare is the only thing that matters (Sumner, 1999, p. 184; A. Moore & Crisp, 1996, p. 598; Kagan, 1998, p. 48; Shaver, 2004, p. 237). One of the most important contributions of social choice theory has been to provide a more precise formulation and axiomatic characterization of this doctrine using Sen's (1970a) framework of social welfare functionals.

A social welfare functional is a mapping which assigns a social preference ordering of alternatives to each profile of real-valued utility functions in its domain. A social welfare functional is welfarist just in case the ordering it assigns to any profile is determined by a single ordering of utility vectors (Gevers, 1979). This means that the social welfare functional ignores all non-welfare features of the alternatives as well as the particularities of each profile. One of the most fundamental results in social choice theory is the *welfarism theorem*, which, given an unrestricted domain of utility profiles, characterizes welfarism in terms of a Pareto Indifference axiom and a utility-theoretic version of the Independence of Irrelevant Alternatives (Bossert and Weymark, 2004, Theorem 2.2; see also d'Aspremont and Gevers, 1977; Hammond, 1979; Weymark, 1998).

This formulation does capture one important way of being a welfarist. But it also seems possible for a social planner to be welfarist, in the rough sense stated above, even if her choices are not rationalizable as maximizing a single binary social preference relation—let alone an ordering—and thus not accurately modeled by the social welfare functional framework. For example, some believe that in a choice between (a) saving one person from a severe impairment and (b) saving a much larger number of people from a moderate impairment, we ought to choose b; and in a choice between b and (c) saving some even larger number of people from a slight impairment, we ought to choose c; but in a choice between a and c, we ought to choose a, no matter how many people would slightly better off in c (Kamm, 2007, p. 485, Voorhoeve, 2014). Such a pattern of choices is not rationalizable as maximizing any binary relation. But it nonetheless seems compatible with welfarism, in the intuitive sense that the acceptable choices from any set of alternatives depends only on how well off each individual would be in those alternatives. (For many other examples, see Boonin-Vail, 1996; Katz, 2011; Otsuka, 2018; Temkin, 2012; Tungodden and Vallentyne, 2005.)

Unfortunately, the extant literature does not seem to contain a characterization of welfarism in this more general sense. Indeed, while some have weakened the requirement that social preferences be an *ordering*, the resulting characterizations of welfarism, even for a single profile, still require social preferences to be transitive (Weymark, 2017; the need for transitivity is also emphasized by Fleurbaey, Tungodden, and Chang, 2003). This paper addresses this gap, by defining welfarism in a choice-functional framework and providing an axiomatic characterization of welfarism so defined, which does not require social choice functions to be rationalizable. This characterization also has the virtue of being compatible with any view about the structure of individual welfare (e.g., it does not require real-valued utilities). It is, I believe, the most general axiomatic characterization of welfarism to date.

We begin, in section 2, by laying out a novel generalization of Sen (1977)'s framework of *functional collective choice rules*. We then provide a simple characterization of *profile-dependent* welfarism in this framework (section 3), and then extend this characterization to full (i.e., profile-independent) welfarism (section 4). Finally, we distinguish between two kinds of anonymity principles which might be imposed on welfarist choice rules, and provide a choice-functional characterization of anonymous welfarism (section 5).

2 Framework

Let X be a nonempty set of alternatives and $N = \{1, \ldots, n\}$ a nonempty, finite set of individuals. (We do not require the typical assumptions that $|X| \geq 3$ or $|N| \geq 2$.) For each individual $i \in N$, there is a nonempty set $\mathbb{W}_i$ of possible welfare levels for i. These welfare levels can be any objects whatsoever. They could be real numbers, as in the standard framework of Sen (1970a). But they could also be vectors of numbers (as in Chipman, 1960; List, 2004; Sen, 1980), non-numerical "grades" (as in Balinski and Laraki, 2010; Morreau and Weymark, 2016), "dimensioned quantities" of well-being (as in Nebel, 2021, 2022, 2023), or objects of any other kind. And, even if the welfare levels are real-valued, $\mathbb{W}_i$ need not be $\mathbb{R}$, $\mathbb{R}_+$, or $\mathbb{R}_{++}$, as is typically assumed. We also leave open, until section 5, whether different individuals have the same possible welfare levels.

For each individual i, $\mathbb{W}_i^X$ is the set of all *welfare functions* $W_i : X \to \mathbb{W}_i$ for i. A *welfare profile* is an n-tuple of individual welfare functions $W = (W_1, \ldots, W_n)$. We are interested in some nonempty domain $\mathcal{D} \subseteq \prod_{i \in N} \mathbb{W}_i^X$ of possible welfare profiles. Given a welfare profile $W \in \mathcal{D}$ and alternative $x \in X$, $W(x) = (W_1(x), \ldots, W_n(x))$ is x's welfare distribution according to W. (We do not call $W(x)$ a "vector" since these objects need not live in a vector space.)

For any set S, let $\mathcal{F}(S)$ denote the set of all finite, nonempty subsets of S—i.e., *menus* of elements of S. Each $A \in \mathcal{F}(X)$ is a menu of alternatives. A *social choice function* $C : \mathcal{F}(X) \to \mathcal{F}(X)$ takes each menu A of alternatives and returns a nonempty subset $C(A) \subseteq A$ of acceptable choices.

It will be useful to relate some of our axioms and results to well-known properties of choice functions and conditions for rationalizability. We say that $C : \mathcal{F}(X) \to \mathcal{F}(X)$ is rationalized by a binary relation $\succcurlyeq$ on X iff, for all $A \in \mathcal{F}(X)$, $C(A) = \{x \in A \mid x \succcurlyeq y \text{ for all } y \in A\}$. Rationalizability in this sense is equivalent to the conjunction of properties α and γ (Sen, 1971):

Property α If $A \subseteq B$ then $C(B) \cap A \subseteq C(A)$.

Property γ $C(A) \cap C(B) \subseteq C(A \cup B)$.

We call a choice function *fully* rationalizable iff it is rationalized by an ordering. This status is equivalent to Arrow (1959)'s choice axiom—or, equivalently, the conjunction of properties α and β:

Arrow's choice axiom If $A \subseteq B$ and $C(B) \cap A \neq \emptyset$, then $C(B) \cap A = C(A)$.

Property β If $A \subseteq B$ and $x, y \in C(A)$, then $x \in C(B)$ iff $y \in C(B)$.

Indeed, Arrow's choice axiom is equivalent to full rationalizability even for the more general class of choice functions whose domains are closed under finite unions (Hansson, 1968; Le Breton and Weymark, 2011; Suzumura, 1983). Most of our results below would likewise hold under this generalization.

Let $\mathfrak{C}$ denote the set of all choice functions on $\mathcal{F}(X)$. Adapting the terminology of Sen (1976, 1977, 1993), a *functional collective choice rule* (FCCR) is a mapping $\phi : \mathcal{D} \to \mathfrak{C}$ which assigns a social choice function to each welfare profile in its domain. (This simply replaces Sen's domain of *preference* profiles—n-tuples of orderings on X—with one of welfare profiles.) For any profile W, we write C_W for $\phi(W)$.

We can distinguish two levels at which welfarism might be applied (Blackorby, Donaldson, and Weymark, 1990). It might first be applied only *within* each profile, to the social choice function assigned to that profile: that is, for any profile W, the choice function C_W's selection from any menu of alternatives might depend only on the welfare distributions assigned to those alternatives by W. This is *profile-dependent* welfarism. A stronger property applies *across* profiles. It says that there is a single choice function on the set of all menus of welfare distributions which determines the choice function C_W assigned to any profile W. This is welfarism *simpliciter*. We characterize these two ideas in turn.

3 Profile-Dependent Welfarism

For any profile W and subset of alternatives $S \subseteq X$, let

$$\mathbf{W}(S) := \{\, w \in \prod_{i \in N} \mathbb{W}_i \mid w = W(x) \text{ for some } x \in S \,\}$$

denote the set of welfare distributions attainable by alternatives in S in W. According to

Profile-Dependent Welfarism For any profile $W \in \mathcal{D}$, there is a unique choice function $C^*_W : \mathcal{F}(\mathbf{W}(X)) \to \mathcal{F}(\mathbf{W}(X))$ such that, for all $A \in \mathcal{F}(X)$ and $x \in A$, $x \in C_W(A)$ iff $W(x) \in C^*_W(\mathbf{W}(A))$.

Profile-Dependent Welfarism captures the idea that, within any given profile, the social choice from any menu of alternatives should be fully determined by their welfare distributions in that profile; non-welfare features of the alternatives can be ignored. However, it allows the distributive choice function to vary between profiles.

We first show Profile-Dependent Welfarism to be equivalent to the following condition:

Intraprofile Neutrality For any $W \in \mathcal{D}$ and $A, B \in \mathcal{F}(X)$, if there is a surjection $f : A \to B$ such that $W(x) = W(f(x))$ for all $x \in A$, then, for all $x \in A$, $x \in C_W(A)$ iff $f(x) \in C_W(B)$.

The function f is only required to be surjective because we want the condition to hold even when $|A| > |B|$, as long as the same welfare distributions are attainable in both menus.

Theorem 1. *An FCCR ϕ satisfies Profile-Dependent Welfarism iff it satisfies Intraprofile Neutrality.*

Proof. Suppose that ϕ satisfies Intraprofile Neutrality. For each $W \in \mathcal{D}$, define C^*_W as follows: for any menu of welfare distributions $A^* \in \mathcal{F}(\mathbf{W}(X))$ and $w \in A^*$, $w \in C^*_W(A)$ iff there is some menu of alternatives $A \in \mathcal{F}(X)$ and $x \in A$ such that $\mathbf{W}(A) = A^*$, $W(x) = w$, and $x \in C_W(A)$. For any $A^* \in \mathcal{F}(\mathbf{W}(X))$, there must be some $A \in \mathcal{F}(X)$ such that $\mathbf{W}(A) = A^*$, and there must be some $x \in A$ such that $x \in C_W(A)$, in which case $W(x) \in C^*_W(A^*)$. Thus, C^*_W always returns a nonempty choice set, so it is a choice function.

Take any $A \in \mathcal{F}(X)$ and $a \in A$. Clearly $a \in C_W(A)$ implies $W(a) \in C^*_W(\mathbf{W}(A))$. For the converse implication, suppose $W(a) \in C^*_W(\mathbf{W}(A))$. Then there must be some $B \in \mathcal{F}(X)$ and $b \in B$ such that $\mathbf{W}(B) = \mathbf{W}(A)$, $W(a) = W(b)$, and $b \in C_W(B)$. Now either $|A| \geq |B|$ or $|B| \geq |A|$. In the former case there is a surjection $f : A \to B$ such that $W(x) = W(f(x))$ for every $x \in A$ and $f(a) = b$; in the latter case, there is a surjection $g : B \to A$ such that $W(y) = W(g(y))$ for every $y \in B$ and $g(b) = a$. Either way, $b \in C_W(B)$ implies $a \in C_W(A)$, by Intraprofile Neutrality. Thus, for all $A \in \mathcal{F}(X)$ and $x \in A$, $x \in C_W(A)$ iff $W(x) \in C^*_W(\mathbf{W}(A))$.

To see that C^*_W must be unique, take any other choice function C^{**}_W on $\mathcal{F}(\mathbf{W}(X))$ such that $x \in C_W(A)$ iff $W(x) \in C^{**}_W(\mathbf{W}(A))$ for all $A \in \mathcal{F}(X)$ and $x \in A$. These choice functions are distinct only if, for some $A \in \mathcal{F}(X)$, $C^{**}_W(\mathbf{W}(A)) \neq C^*_W(\mathbf{W}(A))$. This is impossible given our result that, for all $A \in \mathcal{F}(X)$ and $x \in A$, $x \in C_W(A)$ iff $W(x) \in C^*_W(\mathbf{W}(A))$.

Suppose next that ϕ satisfies Profile-Dependent Welfarism: there is a choice function $C^*_W : \mathcal{F}(\mathbf{W}(X)) \to \mathcal{F}(\mathbf{W}(X))$ such that, for all $A \in \mathcal{F}(X)$ and $x \in A$, $x \in C_W(A)$ iff

$W(x) \in C^*_W(\mathbf{W}(A))$. Take any $A, B \in \mathcal{F}(X)$ and $W \in \mathcal{D}$ for which there is a surjection $f : A \to B$ such that $W(x) = W(f(x))$ for all $x \in A$. For any $x \in A$, $x \in C_W(A)$ iff $W(x) \in C^*_W(\mathbf{W}(A))$, and $f(x) \in C_W(B)$ iff $W(f(x)) \in C^*_W(\mathbf{W}(B))$. Since $\mathbf{W}(A) = \mathbf{W}(B)$, $W(x) \in C^*_W(\mathbf{W}(A))$ iff $W(f(x)) \in C^*_W(\mathbf{W}(B))$. Thus $x \in C_W(A)$ iff $f(x) \in C_W(B)$, so Intraprofile Neutrality is satisfied. □

Intraprofile Neutrality is, in turn, equivalent to the conjunction of three independent conditions. The first is a choice-functional variation on Pareto indifference. It says that the social choice function assigned to any given profile cannot discriminate between two alternatives which have the same welfare distribution, in the sense that either both or neither are acceptable choices from any menu to which they both belong:

Pareto Indiscriminability For any $W \in \mathcal{D}$, $A \in \mathcal{F}(X)$ and $x, y \in A$, if $W(x) = W(y)$, then $x \in C_W(A)$ iff $y \in C_W(A)$.

The second condition is a restriction of property α to menus which have the same attainable welfare distributions. To motivate this idea, say that an alternative is *redundant* on a menu if there is some other alternative on that menu with the same welfare distribution. Redundant Contraction captures the idea that removing redundant alternatives can't make an initially acceptable choice suddenly unacceptable:

Redundant Contraction For any $W \in \mathcal{D}$ and $A, B \in \mathcal{F}(X)$, if $\mathbf{W}(A) = \mathbf{W}(B)$ and $A \subseteq B$, then $C_W(B) \cap A \subseteq C_W(A)$.

The third condition expresses the "converse" idea that if we expand a menu to include new alternatives whose welfare distributions were already attainable in the original menu, this should not make any initially acceptable alternatives suddenly unacceptable:

Redundant Expansion For any $W \in \mathcal{D}$ and $A, B \in \mathcal{F}(X)$, if $\mathbf{W}(A) = \mathbf{W}(B)$ and $A \subseteq B$, then $C_W(A) \subseteq C_W(B)$.

These three conditions are jointly equivalent to Intraprofile Neutrality:

Theorem 2. *An FCCR satisfies Intraprofile Neutrality iff it satisfies Pareto Indiscriminability, Redundant Expansion, and Redundant Contraction.*

Proof. Suppose first that ϕ satisfies Pareto Indiscriminability, Redundant Expansion, and Redundant Contraction. Take any profile $W \in \mathcal{D}$ and any $A, B \in \mathcal{F}(X)$ for which there's a surjection $f : A \to B$ such that $W(x) = W(f(x))$ for all $x \in A$. Clearly $\mathbf{W}(A) = \mathbf{W}(B)$.

Suppose that $x \in C_W(A)$. Since $A \subseteq A \cup B$, Redundant Expansion implies that $x \in C_W(A \cup B)$. Pareto Indiscriminability then implies that $f(x) \in C_W(A \cup B)$. Since $B \subseteq A \cup B$, Redundant Contraction implies that $f(x) \in C_W(B)$. By exactly similar reasoning, if $f(x) \in C_W(B)$, then $f(x) \in C_W(A \cup B)$ by Redundant Expansion, which implies $x \in C_W(A \cup B)$ by Pareto Indiscriminability, and therefore $x \in C_W(A)$ by Redundant Contraction. Thus, Intraprofile Neutrality is satisfied.

Suppose next that ϕ satisfies Intraprofile Neutrality. To derive Pareto Indiscriminability, take any $W \in \mathcal{D}$, $A \in \mathcal{F}(X)$, and $a, b \in A$ such that $W(a) = W(b)$. Clearly there is a surjection $f : A \to A$ such that $W(x) = W(f(x))$ for all $x \in A$, and $f(a) = b$. Thus by Intraprofile Neutrality, $a \in C_W(A)$ iff $b \in C_W(A)$, so Pareto Indiscriminability is satisfied.

Now take any $W \in \mathcal{D}$ and $A, B \in \mathcal{F}(X)$ such that $\mathbf{W}(A) = \mathbf{W}(B)$ and $A \subseteq B$. There must be a surjection $f : B \to A$ such that $W(x) = W(f(x))$ for all $x \in B$ and $f(x) = x$ for all $x \in A$. Intraprofile Neutrality then immediately implies that for any $x \in A$, $x \in C_W(A)$ iff $x \in C_W(B)$. This establishes both Redundant Expansion and Redundant Contraction. □

These three axioms—Pareto Indiscriminability, Redundant Expansion, and Redundant Contraction—are independent in the sense that no two of them entail the third, given a set of at least three alternatives. To see this, let $X = \{a, b, c\}$ and $W(a) = W(b) \neq W(c)$. Consider the choice functions depicted in Table 1. There are four non-singleton menus in $\mathcal{F}(X)$, one in each row. Each column depicts a choice function that violates the axiom listed while satisfying the other two. The set in each cell is the value of the corresponding choice function (column) in the corresponding menu (row). The key feature that makes these counterexamples possible is that Redundant Expansion and Redundant Contraction are restricted to menus with the same sets of attainable welfare distributions, which no other non-singleton menu has in common with $\{a, b\}$. For example, in the violation of Pareto Indiscriminability, while Redundant Expansion requires $C_W(\{a, b\}) = \{a, b\}$ (since $a \in C_W(\{a\}) \cap \{a, b\}$ and $b \in C_W(\{b\}) \cap \{a, b\}$ and $\mathbf{W}(\{a\}) = \mathbf{W}(\{b\}) = \mathbf{W}(\{a, b\})$), it does not require $b \in C_W(\{a, b, c\})$, since $\mathbf{W}(\{a, b\}) \neq \mathbf{W}(\{a, b, c\})$.

None of the choices rules in Table 1 is fully rationalizable. For example, property β is violated by the counterexample to Pareto Indiscriminability, α by the counterexample to Redundant Contraction, and both β and γ by the counterexample to Redundant Expansion. Indeed, Redundant Contraction and Redundant Expansion together imply a condition that resembles Arrow's choice axiom—namely, that if $\mathbf{W}(A) = \mathbf{W}(B)$ and $A \subseteq B$, then $C_W(B) \cap A = C_W(A)$. But, crucially, our axioms do not imply Arrow's because they are restricted to menus which have the same attainable welfare distributions. (Arrow's axiom, on the

Menu	Pareto Indiscriminability	Redundant Contraction	Redundant Expansion
$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$
$\{b, c\}$	$\{c\}$	$\{c\}$	$\{b, c\}$
$\{a, c\}$	$\{a, c\}$	$\{a\}$	$\{a, c\}$
$\{a, b, c\}$	$\{a, c\}$	$\{a, b, c\}$	$\{c\}$

Table 1: Counterexamples to each of Pareto Indiscriminability, Redundant Contraction, Redundant Expansion, where $W(a) = W(b) \neq W(c)$.

other hand, does imply Redundant Contraction and Redundant Expansion, given Pareto Indiscriminability.)

This means that our axioms are compatible with nonrationalizability. For example, suppose that $X = \{a, b, c\}$ and $W(a) \neq W(b) \neq W(c) \neq W(a)$. Then all of the choice functions depicted in Table 1 would be compatible with Pareto Indiscriminability, Redundant Expansion, and Redundant Contraction, and therefore with Intraprofile Neutrality and Profile-Dependent Welfarism. We give a more informative example in section 5.

4 Welfarism

Let $\Omega := \{ w \in \prod_{i \in N} \mathbb{W}_i \mid w = W(x) \text{ for some } W \in \mathcal{D}, x \in X \}$ denote the set of all welfare distributions attainable across all profiles. Let

$$\mathcal{D}^* := \{ A^* \in \mathcal{F}(\Omega) \mid A^* = \mathbf{W}(A) \text{ for some } W \in \mathcal{D}, A \in \mathcal{F}(X) \}$$

denote the set of all menus of welfare distributions which are attainable by some menu of alternatives in some profile or other. According to

Welfarism There is a unique choice function $C^* : \mathcal{D}^* \to \mathcal{D}^*$ such that, for all $W \in \mathcal{D}$, $A \in \mathcal{F}(X)$, and $x \in A$, $x \in C_W(A)$ iff $W(x) \in C^*(\mathbf{W}(A))$.

We call C^* the *distributive choice function* associated with the FCCR ϕ.

Welfarism is equivalent to the following strengthening of Intraprofile Neutrality:

Interprofile Neutrality For any $A, B \in \mathcal{F}(X)$ and $W, W' \in \mathcal{D}$, if there is a surjection $f : A \to B$ such that $W(x) = W'(f(x))$ for all $x \in A$, then $x \in C_W(A)$ iff $f(x) \in C_{W'}(B)$ for all $x \in A$.

Theorem 3. *An FCCR satisfies Welfarism iff it satisfies Interprofile Neutrality.*

Proof. Suppose that ϕ satisfies Interprofile Neutrality. It therefore satisfies Intraprofile Neutrality (by letting $W = W'$) and thus, by Theorem 1, Profile-Dependent Welfarism. We can then define the distributive choice function C^* as the union of all the profile-dependent choice functions C^*_W across $\mathcal{D}$: that is, $C^* \coloneqq \bigcup_{W \in \mathcal{D}} C^*_W$.

Take any $A^* \in \mathcal{D}^*$. There must be some $W \in \mathcal{D}$ such that $A^* \in \mathcal{F}(\mathbf{W}(X))$. We show that, for any other $W' \in \mathcal{D}$ such that $A^* \in \mathcal{F}(\mathbf{W}'(X))$, $C^*_W(A^*) = C^*_{W'}(A^*)$. There must be some $A, B \in \mathcal{F}(X)$ such that $\mathbf{W}(A) = A^* = \mathbf{W}'(B)$. Either $|A| \geq |B|$ or $|B| \geq |A|$. So there is either a surjection $f : A \to B$ or a surjection $g : B \to A$ such that $W(x) = W'(f(x))$ for all $x \in A$ or $W'(y) = W(g(y))$ for all $y \in B$. Interprofile Neutrality then implies that $x \in C_W(A)$ iff $f(x) \in C_{W'}(B)$ for all $x \in A$, or $y \in C_{W'}(B)$ iff $g(y) \in C_W(A)$ for all $y \in B$. Either way, $\mathbf{W}(C_W(A)) = \mathbf{W}'(C_{W'}(B))$, and thus $C^*_W(A^*) = C^*_{W'}(A^*)$ by Profile-Dependent Welfarism.

It follows that $C^*(A^*)$ exists and is a unique, nonempty subset of A^* for all $A^* \in \mathcal{D}^*$. Thus C^* is a choice function on $\mathcal{D}^*$. Profile-Dependent Welfarism implies that for all $W \in \mathcal{D}$, $A \in \mathcal{F}(X)$, and $x \in A$, $x \in C_W(A)$ iff $W(x) \in C^*_W(\mathbf{W}(A))$. Thus, for all $W \in \mathcal{D}$, $A \in \mathcal{F}(X)$, and $x \in A$, $x \in C_W(A)$ iff $W(x) \in C^*(\mathbf{W}(A))$, so Welfarism is satisfied.

Suppose next that ϕ satisfies Welfarism. Now take any $A, B \in \mathcal{F}(X)$ and $W, W' \in \mathcal{D}$ for which there is a surjection $f : A \to B$ such that $W(x) = W'(f(x))$ for all $x \in A$. For any $x \in A$, $x \in C_W(A)$ iff $W(x) \in C^*(\mathbf{W}(A))$ by Welfarism. Since $\mathbf{W}(A) = \mathbf{W}'(B)$, $W(x) \in C^*(\mathbf{W}(A))$ iff $W'(f(x)) \in C^*(\mathbf{W}'(B))$. And $f(x) \in C_{W'}(B)$ iff $W'(f(x)) \in C^*(\mathbf{W}(B))$ by Welfarism. Thus Interprofile Neutrality is satisfied.

□

Interprofile Neutrality is, in turn, equivalent—under certain circumstances—to the conjunction of Intraprofile Neutrality and

Independence of Irrelevant Alternatives For all $W, W' \in \mathcal{D}$ and $A \in \mathcal{F}(X)$, if $W(x) = W'(x)$ for all $x \in A$, then $C_W(A) = C_{W'}(A)$.

Our proof of this equivalence involves two assumptions. First, we assume that the domain is unrestricted in the following sense:

Unrestricted Domain For any $A \in \mathcal{F}(X)$, $A^* \in \mathcal{D}^*$, and $g : A \to A^*$, there is a $W \in \mathcal{D}$ such that $W(x) = g(x)$ for all $x \in A$.

(Here g need not be either surjective or injective, so we can have $|A| > |A^*|$ or $|A| < |A^*|$.) Even when $\mathbb{W}_i = \mathbb{R}$ for all $i \in N$, Unrestricted Domain is considerably weaker than the usual axiom of that name. It is compatible, for example, with certain people's utilities always being of the opposite sign, or always being integers.

Second, we need an assumption about the number of alternatives. It is simplest to assume that X is infinite. We make this assumption in Theorem 4 below. However, the result would still hold if X were finite, so long as there are more alternatives than welfare distributions in Ω. This more complicated version of the theorem is proved in the appendix.

Theorem 4. *If an FCCR ϕ satisfies Unrestricted Domain and $|X| = \infty$, then ϕ satisfies Interprofile Neutrality iff ϕ satisfies Pareto Indiscriminability, Redundant Contraction, Redundant Expansion, and Independence of Irrelevant Alternatives.*

Proof. Assume that ϕ satisfies Unrestricted Domain, Pareto Indiscriminability, Redundant Contraction, Redundant Expansion, and Independence of Irrelevant Alternatives. By Theorem 2, ϕ satisfies Intraprofile Neutrality. Take any $A, B \in \mathcal{F}(X)$ and $W, W' \in \mathcal{D}$ for which there is a surjection $f : A \to B$ such that $W(x) = W'(f(x))$ for all $x \in A$. Since $|X| = \infty$ and A and B are finite, we can find some $A' \in \mathcal{F}(X)$ which is disjoint from A and B and some bijection $h : A \to A'$ (thus $|A| = |A'|$). By Unrestricted Domain, there are profiles V and V' such that:

- For all $x \in A$, $V'(h(x)) = V(h(x)) = V(x) = W(x)$, and
- For all $y \in B$, $V'(y) = W'(y)$.

For all $x \in A$, Independence of Irrelevant Alternatives and Intraprofile Neutrality imply (in alternating order) that $x \in C_W(A)$ iff $x \in C_V(A)$ iff $h(x) \in C_V(A')$ iff $h(x) \in C_{V'}(A')$ iff $f(x) \in C_{V'}(B)$ iff $f(x) \in C_{W'}(B)$. Thus, we have $x \in C_W(A)$ iff $f(x) \in C_{W'}(B)$ for all $x \in A$, so Interprofile Neutrality is satisfied.

It is easy to see that Interprofile Neutrality implies Independence of Irrelevant Alternatives and Intraprofile Neutrality and, therefore, Pareto Indiscriminability, Redundant Contraction, and Redundant Expansion. □

The full strength of Unrestricted Domain is not necessary for the equivalence in Theorem 4. For example, suppose there's a nonempty $S \subseteq X$ such that $W(x) = W'(x)$ for all $x \in S$ and $W, W' \in \mathcal{D}$. This violates Unrestricted Domain as long as there are at least two attainable welfare distributions. But clearly the restriction of Interprofile Neutrality to A or B in $\mathcal{F}(S)$ would hold as long as Intraprofile Neutrality is satisfied. We could of course weaken Unrestricted Domain to accommodate this sort of possibility, as long as $X \setminus S$ is either infinite, bigger than Ω, or empty. One question for further research is how much further Unrestricted Domain can be weakened while maintaining the necessary equivalence of Interprofile Neutrality to the conjunction of Intraprofile Neutrality and Independence of Irrelevant Alternatives, given a suitable number of alternatives.

5 Anonymity and Rationalizability

A welfarist believes that welfare is the only thing that matters. This is precisified by our axiomatic characterization above. But we might also believe that it doesn't matter *who* has what welfare. This does not follow from Welfarism as we have defined it. Many welfarists will want to impose some further constraint to capture a requirement of impartiality between individuals.

Anonymity principles are meant to reflect this idea of impartiality. There are two ways in which a distributive choice function C^* might be anonymous. The first is that it may be unable to discriminate between welfare distributions related by a permutation of individuals:

Anonymous Indiscriminability For all $A^* \in \mathcal{D}^*$ and $w, v \in A^*$, if there is a permutation $\sigma : N \to N$ such that $w_i = v_{\sigma(i)}$ for all $i \in N$, then $w \in C^*(A^*)$ iff $v \in C^*(A^*)$.

Many social choice principles which are naturally modelled in the FCCR framework, however, are incompatible with Anonymous Indiscriminability. For example, as mentioned in section 1, many people think we ought to save a single person from severe harm (such as torture or death) rather than any number of people from a slight impairment (such as a headache). Consider the distributions in Table 2, where welfare levels are represented by real numbers. Suppose that losing 99 units of welfare corresponds to a severe harm whereas losing 1 unit corresponds to a merely slight harm. Then, on the view under consideration, we ought to choose w rather than v, in violation of Anonymous Indiscriminability (Brown, 2020; Parfit, 2003; Voorhoeve, 2014).

	Person 1	Person 2	…	Person 100
w	100	1	…	99
v	1	2	…	100

Table 2: Violation of Anonymous Indiscriminability

This violation of Anonymous Indiscriminability, however objectionable it may be, doesn't seem to involve any failure of impartiality. A social planner who chooses w rather than v

needn't care more about *person* 1 than anyone else; rather, they may simply care more about preventing severe harms, whomever might befall them, than preventing any number of minor ones. In particular, C^* may satisfy the following condition, which really does seem a requirement of impartiality:

Anonymous Invariance For all $A^*, B^* \in \mathcal{D}^*$, if there is a permutation $\sigma : N \to N$ and a bijection $f : A^* \to B^*$ such that $w_i = f(w)_{\sigma(i)}$ for all $i \in N$ and $w \in A^*$, then $w \in C^*(A^*)$ iff $f(w) \in C^*(B^*)$ for all $w \in A^*$.

For example, if we chose w rather than v from Table 2 but v' rather than w' from Table 3, that would violate Anonymous Invariance.

	Person 1	Person 2	…	Person 100
w'	1	100	…	99
v'	2	1	…	100

Table 3: Violation of Anonymous Invariance

In order to derive Anonymous Invariance, we need another assumption about the domain of our FCCR:

Interpersonal Richness For any profile $W \in \mathcal{D}$ and permutation $\sigma : N \to N$, there is a profile $W' \in \mathcal{D}$ such that $W_i = W'_{\sigma(i)}$ for every $i \in N$.

This is not already implied by Unrestricted Domain, which is compatible with different individuals having no possible welfare levels in common. Interpersonal Richness rules this out.

Given Welfarism and Interpersonal Richness, Anonymous Invariance is equivalent to imposing the following anonymity condition on our FCCR:

Interprofile Anonymity For all $W, W' \in \mathcal{D}$, if there is a permutation $\sigma : N \to N$ such that $W_i = W'_{\sigma}(i)$ for all $i \in N$, then $C_W = C_{W'}$.

Theorem 5. *If an FCCR ϕ satisfies Welfarism and Interpersonal Richness, then ϕ satisfies Interprofile Anonymity iff its distributive choice function C^* satisfies Anonymous Invariance.*

Proof. Assume Interprofile Anonymity, Welfarism, and Interpersonal Richness. Take any $A^*, B^* \in \mathcal{D}^*$ for which there is a permutation $\sigma : N \to N$ and a bijection $f : A^* \to B^*$ such that $w_i = f(w)_{\sigma(i)}$ for all $i \in N, u \in A^*$. Since $A^* \in \mathcal{D}^*$, there must be some $A \in \mathcal{F}(X)$ and

$W \in \mathcal{D}$ such that $A^* = \mathbf{W}(A)$. By Interpersonal Richness, there is also a profile $W' \in \mathcal{D}$ such that $W_i = W'_{\sigma(i)}$ for all $i \in N$.

Take any $w \in A^*$. There must be some $a \in A$ such that $W(a) = w$. Welfarism implies that $w \in C^*(A^*)$ iff $a \in C_W(A)$. Interprofile Anonymity then implies that $a \in C_W(A)$ iff $a \in C_{W'}(A)$. Since $W'(a) = f(w)$, Welfarism then implies that $a \in C_{W'}(A)$ iff $f(w) \in C^*(B^*)$. Thus, Anonymous Invariance is satisfied.

It is easy to see that Anonymous Invariance and Welfarism imply Interprofile Anonymity. □

In light of Theorem 5, I call an FCCR *anonymously welfarist* iff it satisfies Welfarism and Interprofile Anonymity. A simple example of a nonrationalizable but anonymously welfarist FCCR is the *leximax loss* rule. Assume $\mathbb{W}_i = \mathbb{R}$ for all $i \in N$. For any $W \in \mathcal{D}$, $A \in \mathcal{F}(X)$, $x \in A$, and $i \in N$, let $L_i^A(W(x)) \coloneqq \max_{y \in A} W_i(y) - W_i(x)$ denote the magnitude of i's "loss" in x relative to her best alternative in A, according to profile W. The *loss vector* of x in A, according to W, is $L^A(W(x)) = (L_1^A(W(x)), \ldots, L_n^A(W(x)))$. For any $u \in \mathbb{R}^n$, let $u_{()} = (u_{(1)}, \ldots, u_{(n)})$ be a permutation of u such that $u_{(i)} \geq u_{(i+1)}$ for all $i \in \{1, \ldots, n-1\}$. The *leximax loss* FCCR says that, for all $W \in \mathcal{D}$, $A \in \mathcal{F}(X)$, and $x \in A$, $x \in C_W(A)$ iff, for all $y \in A$, either $L_{()}^A(W(x)) = L_{()}^A(W(y))$ or there exists $j \in N$ such that $L_{(i)}^A(W(x)) = L_{(i)}^A(W(y))$ for all $i < j$ and $L_{(j)}^A(W(x)) < L_{(j)}^A(W(y))$.

This FCCR satisfies Welfarism. Where L^{A^*} is defined in the obvious way, the corresponding distributive choice function says that, for all $A^* \in \mathcal{D}^*$ and $w \in A^*$, $w \in C^*(A^*)$ iff, for all $v \in A^*$, either $L_{()}^{A^*}(w) = L_{()}^{A^*}(v)$ or there exists $j \in N$ such that $L_{(i)}^{A^*}(w) = L_{(i)}^{A^*}(v)$ for all $i < j$ and $L_{(j)}^{A^*}(w) < L_{(j)}^{A^*}(v)$. However, this choice function is not rationalizable, as is illustrated by Table 4. There the leximax loss rule has $C^*(\{u, v, w\}) = \{u, v, w\}$, but $C^*(\{u, v\}) = \{u\}$, $C^*(\{v, w\}) = \{v\}$, and $C^*(\{w, u\}) = \{w\}$, in violation of property α.

	Person 1	Person 2	Person 3
u	1	2	3
v	2	3	1
w	3	1	2

Table 4: Nonrationalizability of Leximax Loss

The leximax loss rule satisfies Anonymous Invariance but violates Anonymous Indiscriminability. The relationship between Anonymous Invariance, Anonymous Indiscriminability, and rationalizability is summarized by the following result:

Theorem 6. *Given Interpersonal Richness, if a distributive choice function C^* is fully rationalizable, then C^* satisfies Anonymous Indiscriminability iff C^* satisfies Anonymous Invariance.*

Proof. Assume Interpersonal Richness, Anonymous Invariance, and that C^* is fully rationalizable. C^* therefore satisfies properties α and β and, equivalently, Arrow's choice axiom.

Take any permutation $\sigma : N \to N$. Since N is finite, σ is the product of finitely many transpositions $\tau_1, \ldots, \tau_m$ (a transposition is a permutation that swaps exactly two elements). Take any $w^0 \in \mathcal{D}^*$ and let $w^j = \tau_j(w^{j-1})$ for all $j \in \{ 1, \ldots, m \}$, so that $w^m = \sigma(w^0)$. All of these vectors are in $\mathcal{D}^*$ by Interpersonal Richness. We show that whenever $w^0, w^m \in B^*$ for any $B^* \in \mathcal{D}^*$, $w^0 \in C^*(B^*)$ iff $w^m \in C^*(B^*)$.

Anonymous Invariance implies that $C^*(\{ w^{j-1}, w^j \}) = \{ w^{j-1}, w^j \}$ for all $j \in \{ 1, \ldots, m \}$. Property β implies that $C^*(\{ w^0, w^1, \ldots w^m \}) = \{ w^0, w^1, \ldots w^m \}$. Property α implies that $C^*(\{ w^0, w^m \}) = \{ w^0, w^m \}$. β then implies that whenever $w^0, w^m \in B^*$ for any $B^* \in \mathcal{D}^*$, $w^0 \in C^*(B^*)$ iff $w^m \in C^*(B^*)$. Therefore, Anonymous Indiscriminability is satisfied.

Next assume Anonymous Indiscriminability instead of Anonymous Invariance. Take any $A^*, B^* \in \mathcal{D}^*$ such that, for some permutation $\sigma : N \to N$ and bijection $f : A^* \to B^*$, $w_i = f(w)_{\sigma(i)}$ for all $i \in N$ and $w \in A^*$. Anonymous Indiscriminability implies that $C^*(A^* \cup B^*) \cap A^* = \emptyset$ iff $C^*(A^* \cup B^*) \cap B^* = \emptyset$. Thus $C^*(A^* \cup B^*) \cap A^* = C^*(A^*)$ and $C^*(A^* \cup B^*) \cap B^* = C^*(B^*)$ by Arrow's choice axiom. Anonymous Indiscriminability also implies that, for any $w \in A^*$, $w \in C^*(A^* \cup B^*)$ iff $f(w) \in C^*(A^* \cup B^*)$. Thus we have $w \in C^*(A^*)$ iff $w \in C^*(A^* \cup B^*)$ iff $f(w) \in C^*(A^* \cup B^*)$ iff $f(w) \in C^*(B^*)$, as required by Anonymous Invariance. □

We have already seen how a distributive choice function can satisfy Anonymous Invariance while violating Anonymous Indiscriminability. Interestingly, it is also possible, in the absence of rationalizability, to satisfy Anonymous Indiscriminability while violating Anonymous Invariance. Suppose for example that $N = \{ 1, 2 \}$ and $\mathbb{W}_1 = \mathbb{W}_2 = \{ 0, 1, 2 \}$. Let $u = (2, 0)$, $v = (0, 2)$, and $w = (1, 1)$. The choice functions in Table 5 all satisfy Anonymous Indiscriminability but violate Anonymous Invariance. The choice function in the leftmost column violates property β, the middle one violates α, and the one on the right violates γ and β. Unlike the leximax loss rule's violation of Anonymous Indiscriminability, these violations of Anonymous Invariance seem inexplicable from an impartial perspective.

This confirms our suspicion that Anonymous Indiscriminability does not, on its own, capture a fundamental commitment to impartiality; it seems better regarded as a *consequence* of impartiality on the assumption of full rationalizability.

Menu	β	α	γ,β
$\{u,v\}$	$\{u,v\}$	$\{u,v\}$	$\{u,v\}$
$\{v,w\}$	$\{v\}$	$\{w\}$	$\{v,w\}$
$\{u,w\}$	$\{u,w\}$	$\{u\}$	$\{w\}$
$\{u,v,w\}$	$\{u,v\}$	$\{u,v,w\}$	$\{w\}$

Table 5: Anonymous Indiscriminability without Anonymous Invariance

Theorem 6 also sheds some light on requirements of impartiality in more standard, "relational" frameworks for social welfare evaluation. For example, Blackorby, Bossert, and Donaldson (2005b, ch. 7) explore a framework of *social decision functionals*, which assign a (possibly incomplete) quasiordering to each profile of real-valued utility functions in some domain. They require the functional to be welfarist in the sense that the comparison of two alternatives in any profile is determined by a single quasiordering of utility vectors. Our results suggest that, in such a framework, the analogue of Interprofile Anonymity will not force all permutations of a utility vector to be equally good; it will only require the quasiordering of utility vectors to be invariant to common permutations, so that for any vectors u and v and permutation of individuals σ, u is at least as good as v iff $\sigma(u)$ is at least as good as $\sigma(v)$. An example of a social decision functional which satisfies only the latter condition is the strong Pareto rule axiomatized, in an Arrovian setting, by Weymark (1984). The difference between these conditions is especially important in settings with infinite populations, where, given a suitable set of utility vectors, only the anonymity-as-invariance condition is compatible with the strong (or even weak) Pareto principle (Asheim, d'Aspremont, and Banerjee, 2010).

6 Conclusion

We have seen that collective choice rules can be, in a natural sense, welfarist—indeed, anonymously welfarist—even if their prescriptions are not rationalizable by any binary relation, let alone an ordering. An important further lesson of this paper is that the formal characterization of welfarism is not sensitive to the nature or structure of the welfare levels or "utilities" to which alternatives are assigned. In particular, welfarism does not require any commitment to a real-valued representation of welfare, or to each individual having an ordering on the set of alternatives.

This flexibility means that our results can be utilized even by many theorists who reject welfarism. Such theorists can simply reinterpret the sets $\mathbb{W}_i$ to include nonwelfare characteristics of some relevant kind (e.g., which of i's rights are respected, or how deserving or responsible i is). A theorist who accepts our choice-functional "welfarism" axioms when the welfare levels are replaced by these more general properties would be committed to choosing between alternatives on the basis of individuals' characteristics in those alternatives alone, but not necessarily just their welfare characteristics.[1] This doctrine, which might be called *individualism*, would seem acceptable to many critics of welfarism (such as Scanlon, 1998; Sen, 1970b), though not all of them (such as G. E. Moore, 1903). We leave a more thorough exploration of this view for another occasion.

A Variation on Theorem 4 with finitely many alternatives

We first prove the following:

Lemma 7. *If an FCCR satisfies Unrestricted Domain, Intraprofile Neutrality, and Independence of Irrelevant Alternatives, and* $\infty > |X| > |\Omega|$*, then for any* $W, W' \in \mathcal{D}$*, if there is a transposition* $\tau : X \to X$ *such that* $W(x) = W'(\tau(x))$ *for all* $x \in X$*, then for all* $x \in X$*,* $x \in C_W(X)$ *iff* $\tau(x) \in C_{W'}(X)$.

Proof. Assume Unrestricted Domain, Intraprofile Neutrality, and Independence of Irrelevant Alternatives, and take any $W, W' \in \mathcal{D}$ for which there is a transposition $\tau : X \to X$ such that $W(x) = W'(\tau(x))$ for all $x \in X$.

If $|X| \leq 2$, then $|\Omega| = 1$ since $|X| > |\Omega|$, in which case the conclusion follows trivially from Intraprofile Neutrality. So suppose $|X| > 2$. Without loss of generality let $\mathrm{supp}(\tau) = \{ a, b \}$. If $W(a) = W(b)$ then $W = W'$ so $C_W = C_{W'}$. Thus, suppose $W(a) \neq W(b)$. Since $|X| > |\Omega|$, there must be some $c \in X \setminus \{ a, b \}$ such that $W(c) = W(x)$ for some $x \in X \setminus \{ c \}$. Note also that $W(c) = W'(c)$, since $\tau(c) = c$, and that $W'(c) = W'(x)$ for some $x \in X \setminus \{ c \}$.

We now use Unrestricted Domain to construct three profiles $W^1, W^2, W^3 \in \mathcal{D}$:

- $W^1(x) = W(x)$ for all $x \in X \setminus \{ c \}$, and $W^1(c) = W(a)$;

[1] Blackorby, Bossert, and Donaldson (2005a), by contrast, introduce individual non-welfare information only to argue that it is irrelevant, by appealing to a Pareto indifference condition applied to welfare information alone.

- $W^2(x) = W^1(x)$ for all $x \in X \setminus \{ a \}$, and $W^2(a) = W(b)$;
- $W^3(x) = W^2(x)$ for all $x \in X \setminus \{ b \}$, and $W^3(b) = W(a)$.

Intraprofile Neutrality and Independence of Irrelevant Alternatives imply (in alternating order) that $a \in C_W(X)$ iff $a \in C_W(X \setminus \{ c \})$ iff $a \in C_{W^1}(X \setminus \{ c \})$ iff $c \in C_{W^1}(X \setminus \{ a \})$ iff $c \in C_{W^2}(X \setminus \{ a \})$ iff $c \in C_{W^2}(X \setminus \{ b \})$ iff $c \in C_{W^3}(X \setminus \{ b \})$ iff $b \in C_{W^3}(X \setminus \{ c \})$ iff $b \in C_{W'}(X \setminus \{ c \})$ iff $b \in C_{W'}(X)$.

Similarly, they imply (again, in alternating order) that $b \in C_W(X)$ iff $b \in C_W(X \setminus \{ c \})$ iff $b \in C_{W^1}(X \setminus \{ c \})$ iff $b \in C_{W^1}(X \setminus \{ a \})$ iff $b \in C_{W^2}(X \setminus \{ a \})$ iff $a \in C_{W^2}(X \setminus \{ b \})$ iff $a \in C_{W^3}(X \setminus \{ b \})$ iff $a \in C_{W^3}(X \setminus \{ c \})$ iff $a \in C_{W'}(X \setminus \{ c \})$ iff $a \in C_{W'}(X)$.

For any $x \in X \setminus \{ a, b, c \}$ (if there is one), we have $x \in C_W(X)$ iff $x \in C_W(X \setminus \{ c \})$ iff $x \in C_{W^1}(X \setminus \{ c \})$ iff $x \in C_{W^1}(X \setminus \{ a \})$ iff $x \in C_{W^2}(X \setminus \{ a \})$ iff $x \in C_{W^2}(X \setminus \{ b \})$ iff $x \in C_{W^3}(X \setminus \{ b \})$ iff $x \in C_{W^3}(X \setminus \{ c \})$ iff $x \in C_{W'}(X \setminus \{ c \})$ iff $x \in C_{W'}(X)$.

Thus, for any $x \in X \setminus \{ c \}$, we have $x \in C_W(X)$ iff $\tau(x) \in C_{W'}(X)$. Since $W(c) = W(x)$ for some $x \in X \setminus \{ c \}$, Intraprofile Neutrality implies that $c \in C_W(X)$ iff $x \in C_W(X)$ for some such x. We then have $\tau(x) \in C_{W'}(X)$, and since $W'(\tau(x)) = W(x) = W(c) = W'(\tau(c))$, $\tau(c) \in C_{W'}(X)$ as well. □

Theorem 8. *If an FCCR ϕ satisfies Unrestricted Domain and $\infty > |X| > |\Omega|$, then ϕ satisfies Interprofile Neutrality iff ϕ satisfies Intraprofile Neutrality and Independence of Irrelevant Alternatives.*

Proof. As in the proof of Theorem 4, we prove only the right-to-left direction of the biconditional. Assume that ϕ satisfies Unrestricted Domain, Intraprofile Neutrality, and Independence of Irrelevant Alternatives, and that $\infty > |X| > |\Omega|$. Suppose there's a surjection $f : A \to B$ such that $W(x) = W'(f(x))$ for all $x \in A$. Take an arbitrary $a \in A$. Find some subset $A' \subseteq A$ such that $a \in A'$, $|A'| = |B|$, and $\mathbf{W}(A') = \mathbf{W}(A)$. We use Unrestricted Domain to construct profiles V and V' as follows:

- $V(x) = W(x)$ for all $x \in A'$; $V(x) = W(a)$ for all $x \in X \setminus A'$.
- $V'(x) = W'(x)$ for all $x \in B$; $V'(x) = W(a)$ for all $x \in X \setminus B$.

Intraprofile Neutrality and Independence of Irrelevant Alternatives imply (in alternating order) that $a \in C_W(A)$ iff $a \in C_W(A')$ iff $a \in C_V(A')$ iff $a \in C_V(X)$. They also imply (again, in alternating order) that $f(a) \in C_{V'}(X)$ iff $f(a) \in C_{V'}(B)$ iff $f(a) \in C_{W'}(B)$. We therefore need only to show that $a \in C_V(X)$ iff $f(a) \in C_{V'}(X)$.

Observe that the restriction of f to A' is a bijection, and that there is a bijection $g : (X \setminus A') \to (X \setminus B)$ such that $V(x) = V(g(x))$ for all $x \in X \setminus A'$. The union of these bijections is a permutation $\sigma : X \to X$ such that $V(x) = V'(\sigma(x))$ for all $x \in X$. Since X is finite, σ is the product of some transpositions $\tau_1, \ldots, \tau_m$. We can then use Unrestricted Domain to construct profiles $V^1, \ldots, V^{m-1}$ as follows: $V^1(x) = V(\tau_1(x))$ for all $x \in X$; for each $k \in \{2, \ldots, m-1\}$, $V^k(x) = V^{k-1}(\tau_k(x))$. By Lemma 7, we have $a \in C_V(X)$ iff $\tau_1(a) \in C_{V^1}(X)$ iff ... iff $\tau_{m-1}(\ldots(\tau_1(a))\ldots) \in C_{V^{m-1}}(X)$ iff $\tau_m(\ldots(\tau(a))\ldots) = f(a) \in C_{V'}(X)$. □

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